# Black Hole Superpartners and Fixed Scalars

Renata Kallosh and Andrei Linde

*Physics Department, Stanford University, Stanford CA 94305-4060, USA*

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Some bosonic solutions of supergravities admit Killing spinors of unbroken supersymmetry. The anti-Killing spinors of broken supersymmetry can be used to generate the superpartners of stringy black holes. This has a consequent feedback on the metric and the graviphoton. We have found however that the fixed scalars for the black hole superpartners remain the same as for the original black holes. Possible phenomenological implications of this result are discussed.

## I. INTRODUCTION

Recently it was found that black holes serve as attractors for the moduli fields: The values of the moduli near the horizon do not depend on their values far away from the black holes [1,2]. For example, in the theory of dilaton black holes with electric and magnetic charges $q$ and $p$, the dilaton field is given by

$$e^{-2\phi(r)} = \frac{e^{-\phi_0} + \frac{|q|}{r}}{e^{+\phi_0} + \frac{|p|}{r}} \ . \tag{1}$$

Here $\phi_0$ is the value of the dilaton field at infinity, $r$ is the radial coordinate which measures the distance from the horizon. The mass $M$ is related to the electric and magnetic charges as

$$M(\phi_0) = \tfrac{1}{2}(e^{-\phi_0}|p| + e^{\phi_0}|q|) \ . \tag{2}$$

The dilaton field near the horizon approaches the value

$$e^{-2\phi} = \frac{|q|}{|p|} \tag{3}$$

independently of its value at infinity $\phi_0$. This value corresponds to the minimum of the mass $M(\phi)$ (2) with respect to $\phi$. For the non-extreme black holes the situation is similar. The value of $e^{-2\phi}$ at the horizon is not necessarily i equal to $\frac{|q|}{|p|}$. However the mass of the non-extreme black holes with fixed electric and magnetic charges and fixed entropy $S$ has a minimum at the same point (3) as the mass of the extreme ones [3]. Finding such a minimum makes operational sense in the situations where the black hole evaporation is slow and the entropy of a black hole remains constant.

These results are pretty general; they hold for a wide variety of stringy black holes [1–3]. However, it was not known whether the superpartners of these black holes share the same property. In what follows we will show that this is indeed the case: superpartners of stringy black holes are classical solutions with the same electric and magnetic charges, with the same area of horizon, and with the same values of the moduli fields at the horizon.

Thus the universality of superattractors appears to be even more general than we expected.

Our results may be helpful for investigation of scattering on black holes and their superpartners [4]. On the other hand, they might provide a mechanism of moduli fixing in vacuum, and may shed some light on the recently discovered similarity [5] between spontaneous breaking of N= 2 supersymmetry to N = 1 [6] and the behavior of scalar fields near the black hole horizon. The main idea is based on the observation that black holes, their antiparticles (black holes with opposite values of magnetic and electric charges) and their superpartners, which may exist as virtual states in the vacuum, attract the scalar field $\phi$ to the same point (3). This may result in the scalar fields being fixed not only near the horizon of the black holes, but also in the vacuum state containing virtual black holes. A discussion of this interesting but speculative possibility will be contained in Appendix.

## II. FIXED SCALARS FOR BLACK HOLE SUPERPARTNERS

The supersymmetric generalizations of the black hole geometries in the context of pure N=2 supergravity without vector and hyper multiplets was constructed starting from the early 80's [7]. By applying long-range N=2 supergauge transformation on Majumdar-Papapetrou configurations, the set of all superpartners to bosonic multi black holes was exhibited in [7,8].

We will study here what happens with fixed moduli of the bosonic solutions under the transformations required for the construction of the exact superpartners and whether moduli are still stable with the account of supergauge transformations.

The short account of the situation in pure N=2 supergravity [7,8] is the following*. The supersymmetric solutions usually considered have vanishing fermionic fields.

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*This resume follows the one in [9] where we have calculated the norm of the fermionic black hole zero modes.

It was explained in [10], that if non-trivial supersymmetry parameters with the right regularity properties exist, one can generate a whole supermultiplet of solutions. Solutions with non-vanishing fermion fields starting with the purely bosonic ones has been found for extreme Reissner–Nordström solutions solutions of $N = 2$ supergravity in [7,8] by performing supersymmetry transformations with the parameters which converge asymptotically to global supersymmetry parameters. The new solutions are not gauge-equivalent to the original ones (it is not possible to go back to bosonic solutions by using asymptotically vanishing supersymmetry parameters).

There are some non-trivial supersymmetry transformations generated by the Killing spinors of unbroken supersymmetry that leave the original bosonic solution invariant. Only the supersymmetry parameters corresponding to the broken supersymmetries, which we will called "anti-Killing spinors" in [9], [11], generate new non-gauge equivalent solutions.

Consider a solution of the Einstein–Maxwell theory for zero magnetic and positive electric charge

$$ds^2 = V^{-2}dt^2 - V^2 d\vec{x}^2, \quad A_t dt = -\frac{1}{\kappa}V^{-1} dt \ , \tag{4}$$

where $\kappa^2 = 4\pi G$ and the function $V$ is

$$V(\vec{x}) = 1 + \frac{GM}{|\vec{x}\,|} \ , \tag{5}$$

where the horizon of the $s$th black hole is located at $|\vec{x}| = 0$ and its electric charge is $Q_s = G\kappa^{-1}M_s$. This background admits $N = 2$ supergravity Killing spinors [10], *i.e.* a solution of the equation

$$\hat{\nabla}_\mu \epsilon_{(k)} = (\nabla_\mu - \frac{1}{4}\kappa \not{F}\gamma_\mu)\epsilon_{(k)} = 0 \ , \tag{6}$$

where $\not{F} = \gamma^{\mu\nu}F_{\mu\nu}$ and $F_{\mu\nu}$ is the field–strength of the gauge field $A_\mu$ and $\epsilon$ is a Dirac spinor. That solution is given by

$$\epsilon_{(k)} = V^{-1/2}\mathcal{C}_{(k)} \ , \tag{7}$$

where $\mathcal{C}_{(k)}$ is a constant spinor satisfying the condition

$$\gamma_0 \, \mathcal{C}_{(k)} = + \, \mathcal{C}_{(k)}, \qquad \mathcal{C}_{(k)} = \begin{pmatrix} c \\ c \end{pmatrix}, \tag{8}$$

and is given in terms of a complex two-component spinor, $c$. This means that the background given above has one unbroken supersymmetry in $N = 2$ supergravity. The asymptotically constant anti-Killing spinor can be chosen to be, in terms of the Killing spinor,

$$\epsilon^{(\bar{k})} = -i\gamma_5 \epsilon_{(k)} = V^{-1/2}\mathcal{C}^{(\bar{k})} \ , \tag{9}$$

and

$$\gamma_0 \, \mathcal{C}^{(\bar{k})} = - \, \mathcal{C}^{(\bar{k})} \ , \qquad \mathcal{C}^{(\bar{k})} = \begin{pmatrix} c \\ -c \end{pmatrix}. \tag{10}$$

It represents broken supersymmetry of the bosonic solution and is used to generate gravitino [7]:

$$\psi_\mu = \frac{1}{\kappa}\hat{\nabla}_\mu \epsilon^{(\bar{k})} \ . \tag{11}$$

The explicit expression for the gravitino, which solves the field eqs. for gravitino and which is linear in anti-Killing spinor is

$$\psi = \frac{1}{\kappa}V^{-7/2}\partial_{\underline{i}}V\gamma_i \mathcal{C}^{(\bar{k})} \ dt + \frac{1}{\kappa}V^{-3/2}\partial_{\underline{j}}V\gamma_j\gamma_i\mathcal{C}^{(\bar{k})} \ dx^{\underline{i}} \ . \tag{12}$$

Having found gravitino in the linear order (12) one proceeds with the iterative procedure and calculates the feedback of the gravitino on the geometry and the vector field via the supersymmetry transformation generated by the anti-Killing spinor:

$$\delta e_\mu{}^a = -\frac{i\kappa}{2}\left(\bar{\epsilon}^{(\bar{k})}\gamma^a\psi_\mu - \bar{\psi}_\mu\gamma^a\epsilon^{(\bar{k})}\right),$$
$$\delta A_\mu = \frac{i}{2}\left(\bar{\epsilon}^{(\bar{k})}\psi_\mu - \bar{\psi}_\mu\epsilon^{(\bar{k})}\right). \tag{13}$$

This leads to the corrections to the original bosonic solution which is of the second order in Grassmann parameter $c$. This in turn induces additional corrections to the gravitino etc. The full procedure leads to an exact solutions of the full supergravity equations of motion and the series stops after the fourth order in Grassmann numbers. The actual computation was performed in [8] with the help of the algebraic computer program REDUCE. The result schematically can be represented in the form

$$g_{\mu\nu}(\vec{x}, c) = g_{\mu\nu}(\vec{x}) + (c^\dagger\sigma_i c)\,\Delta^i_{\mu\nu}(g) + (c^\dagger c)(c^\dagger c)\,\Delta_{\mu\nu}(g), \tag{14}$$

$$A_\mu(\vec{x}, c) = A_\mu(\vec{x}) + (c^\dagger\sigma_i c)\,\Delta^i_\mu(A) + (c^\dagger c)(c^\dagger c)\,\Delta_\mu(A), \tag{15}$$

where the explicit form of the terms quadratic and quartic in Grassmann numbers $c$ can be found in eqs. (2.34)-(2.36) of the first ref. in [8]. Even when one considers only the near horizon geometry, one still finds that the superpartners have corrections, e.g, the non-diagonal term in the metric as well as the space component of the vector which are proportional to $(c^\dagger\sigma_i c)$ are present, as different from the original geometry which was diagonal and from the vector field which had no space-time component. Also the non-diagonal terms in the 3-dimensional geometry in the quartic order in Grassmann variables are present as different from the original geometry.

The generalization of the procedure [7,8] of the generating exact superpartners of black holes of N=2 supergravity interacting many vector multiplets and hypermultiplets, in principle, can be performed and may also require a considerable effort and most certainly will require the help of a computer. However for our purpose

to understand whether the fixed scalars will be affected and in which way there is a shortcut and we may solve completely the problem with respect to double-extreme black holes [12]. This will be sufficient to understand the effect of the superpartners of any black holes near the horizon, since all supersymmetric black holes near the horizon tend to the double-extreme ones. The crucial point here comes when one looks into the gaugino and hyperino supersymmetric transformation rules. In the first approximation the fermionic fields are absent and we have [†]:

$$\delta\psi_\mu = \mathcal{D}_\mu\epsilon + T_{\mu\nu}^-\gamma^\nu\epsilon \equiv \hat{\mathcal{D}}_\mu\epsilon \ ,$$
$$\delta\lambda^i = i\gamma^\mu\partial_\mu z^i\epsilon + \frac{i}{2}\mathcal{F}_{\mu\nu}^{i-}\gamma^{\mu\nu}\epsilon \ ,$$
$$\delta\zeta_\alpha = i\mathcal{U}_u^\beta\partial_\mu q^u\gamma^\mu\epsilon C_{\alpha\beta} \ , \qquad (16)$$

where $\lambda^i$, $\psi_\mu$ are the chiral gaugino and gravitino fields, $\zeta_\alpha$ is a hyperino. We consider here as in [2] the scalars $q^u$ from the hypermultiplets to be constant for the simplest supersymmetric black hole. The value of this constant does not affect the black hole solution since the scalars from the hypermultiplets do not couple to the vectors. Thus for the constant quaternionic scalars $q^u$ the condition of unbroken supersymmetry

$$\delta\zeta_\alpha = i\mathcal{U}_u^\beta\partial_\mu q^u\gamma^\mu\epsilon C_{\alpha\beta} = 0 \qquad (17)$$

is satisfied without any constraint on the supersymmetry parameters $\epsilon$. The vector multiplets include gaugino, Kahler moduli $z^i$ and the vector fields $\mathcal{F}_{\mu\nu}^{i-}$. Double-extreme black holes have everywhere constant moduli $\partial_\mu z^i = 0$and vanishing vector field strength $\mathcal{F}_{\mu\nu}^{i-} = 0$. This last equation actually means that the central charge is extremized in the moduli space and that the moduli become fixed functions of charges. Thus the unbroken supersymmetry equations for gaugino,

$$\delta\lambda^i = i\gamma^\mu\partial_\mu z^i\epsilon + \frac{i}{2}\mathcal{F}_{\mu\nu}^{i-}\gamma^{\mu\nu}\epsilon = 0 \ , \qquad (18)$$

are satisfied without any constraint on the supersymmetry parameters $\epsilon$. The gravitino supersymmetry transformations even for the double-extreme black hole vanishes only with the Killing spinor satisfying the linear constraint

$$\delta\psi_\mu = \mathcal{D}_\mu\epsilon_{(k)} + T_{\mu\nu}^-\gamma^\nu\epsilon_{(k)} = 0 \ , \qquad (19)$$

since the geometry is that of the extreme Reissner-Nordstrom type with the area-mass formula defined by the charges of the theory. Now we have got enough information to generate the double-extreme black hole superpartners. The first corrections to fermions is given by the anti-Killing spinor:

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[†]In notation of [13], [2]. We skip the spinorial index $A$ in for simplicity.

$$\delta^{(1)}\psi_\mu = \hat{\mathcal{D}}_\mu\epsilon^{(\bar{k})} \ ,$$
$$\delta^{(1)}\lambda^i = i\gamma^\mu\partial_\mu z^i\epsilon^{(\bar{k})} + \frac{i}{2}\mathcal{F}_{\mu\nu}^{i-}\gamma^{\mu\nu}\epsilon^{(\bar{k})} = 0 \ , \qquad (20)$$
$$\delta^{(1)}\zeta_\alpha = i\mathcal{U}_u^\beta\partial_\mu q^u\gamma^\mu\epsilon^{(\bar{k})}C_{\alpha\beta} = 0 \ .$$

The absence of corrections to gaugino and hyperino for the double-extreme black holes follows from their property

$$\partial_\mu z^i = 0 \ , \qquad \mathcal{F}_{\mu\nu}^{i-} = 0 \ , \qquad \partial_\mu q^u = 0 \ . \qquad (21)$$

Thus if one starts with the double-extreme bosonic black hole and performs a long-range N=2 supergauge transformation on fermions, only the gravitino field is generated, the gaugino as well as the hyperino do not appear even in presence of Grassmann numbers $c$, in terms of which gravitino is linear. The complete form of supersymmetry transformation on bosons is

$$\delta e_\mu^{\ a} = -i\bar{\epsilon}^{(\bar{k})}\gamma^a\psi_\mu + c.c. \ , \qquad (22)$$
$$\delta A_\mu^\Lambda = 2\bar{L}^\Lambda\,\bar{\psi}_\mu\,\epsilon + if_i^\Lambda\,\bar{\lambda}^i\,\gamma_\mu\epsilon + c.c. \ , \qquad (23)$$
$$\delta z^i = \bar{\lambda}^i\,\epsilon \ , \qquad (24)$$
$$\delta q^u = \mathcal{U}_\alpha^u\left(\bar{\zeta}^\alpha\epsilon + C_{\alpha\beta}\bar{\zeta}_\beta\epsilon\right) \ . \qquad (25)$$

In the first approximation using (20) one gets the corrections to bosons which are second order in Grassmann numbers:

$$\delta^{(2)}e_\mu^{\ a} = -i\bar{\epsilon}^{(\bar{k})}\gamma^a\delta^{(1)}\psi_\mu + c.c. \ , \qquad (26)$$
$$\delta^{(2)}A_\mu^\Lambda = 2\bar{L}^\Lambda\,\delta^{(1)}\bar{\psi}_\mu\,\epsilon^{(\bar{k})} + c.c. \ , \qquad (27)$$
$$\delta^{(2)}z^i = 0 \ , \qquad (28)$$
$$\delta^{(2)}q^u = 0 \ . \qquad (29)$$

The virbeins and the metric transform via gravitino and will get corrections of the type found in pure supergravity. The vector fields $A_\mu^\Lambda$ get corrections via gravitino. It is important to check what kind of corrections the graviphoton

$$T_{\mu\nu}^- = 2iIm\mathcal{N}_{\Lambda\Sigma}L^\Lambda F^{-\Sigma}$$

has and what happens with the vector fields which are partners of gaugino

$$\mathcal{F}_{\mu\nu}^{i-} = 2iG^{i\bar{j}}Im\mathcal{N}_{\Lambda\Sigma}\bar{f}_{\bar{j}}^\Lambda F^{-\Sigma} \ .$$

Here one has to remember that the graviphoton and the vectors $\mathcal{F}_{\mu\nu}^{i-}$ are orthogonal combinations. The important identity of special geometry explaining this can be found in eq. (54) of [14].

$$Im\mathcal{N}_{\Lambda\Sigma}\bar{f}_{\bar{j}}^\Lambda L^\Sigma = 0 \ . \qquad (30)$$

Using this we find that only the graviphoton has first order corrections quadratic in Grassmann numbers and the partner of gaugino has no such corrections. This is

consistent with the fact that our fixed scalars $z^i$ remain fixed in this order. Thus we get second order corrections for the bosons in the supergravity multiplet

$$\delta^{(2)} e_\mu{}^a = -i\bar{\epsilon}^{(\bar{k})} \gamma^a \hat{\nabla}_\mu \epsilon^{(\bar{k})} + c.c. \ ,$$
$$\delta^{(2)} T_{\mu\nu}^- = 2iIm\mathcal{N}_{\Lambda\Sigma} L^\Lambda \delta^{(2)} F^{-\Sigma} \ , \tag{31}$$

and no second order corrections for the bosons in the vector and hyper multiplets.

$$\delta^{(2)} \mathcal{F}_{\mu\nu}^{i-} = 0 \ ,$$
$$\delta^{(2)} z^i = 0 \ , \tag{32}$$
$$\delta^{(2)} q^u = 0 \ .$$

To solve the problem in the next order we have to look back into the gaugino and hyperino transformation and take into account the presence of fermions and fermionic corrections to bosons. This will give correction terms cubic in Grassmann numbers for fermions. The complete form of the fermionic susy transformations is rather complicated and can be read off from eqs. (8.24)- (8.41) of Ref. [13]. By carefully checking all terms in the full supersymmetry transformation rules we find that the gravitino indeed has corrections of the third order in Grassmann numbers, but neither gaugino nor hyperino have any corrections

$$\delta^{(3)} \psi_\mu \neq 0 \ ,$$
$$\delta^{(3)} \lambda^i = 0 \ , \tag{33}$$
$$\delta^{(3)} \zeta_\alpha = 0 \ .$$

This we can plug back one more time into the bosonic supersymmetry transformations (25) and we get the result we looked for: there are 4-th order terms in the bosons of the supergravity multiplet of the structure displayed in [7,8]

$$\delta^{(4)} e_\mu{}^a \neq 0 \ ,$$
$$\delta^{(4)} T_{\mu\nu}^- \neq 0 \ , \tag{34}$$

and there are no 4-th order corrections for the bosons in the vector and hyper multiplets:

$$\delta^{(4)} \mathcal{F}_{\mu\nu}^{i-} = 0 \ ,$$
$$\delta^{(4)} z^i = 0 \ , \tag{35}$$
$$\delta^{(4)} q^u = 0 \ .$$

Thus indeed the fixed point structure of the near horizon configuration is stable to the supersymmetry transformations which generate the fermionic partners of the supersymmetric black holes. Although the metric and the graviphoton have corrections of the second and fourth order in Grassmann variables and gravitino has terms of the third order, nothing happens with the fixed scalars and their fermionic partners: there are no corrections in all possible orders in Grassmann variables. In particular this means that the moduli $z^i$ in the vector multiplets remain the same functions of charges near the horizon $z^i_{\text{fix}}(p, q)$ for the black hole superpartners as they were for the original black holes.

## III. DISCUSSION

In this paper we have shown that the same mechanism which fixes the scalar fields near the black hole horizon works for the superpartners of the black holes as well. Black holes and their superpartners have the same mass, charges, area of horizon, and entropy. In addition, the scalar fields near the horizon of black holes and of their superpartners reach the same value at the horizon. Thus we have found an interesting universality of the properties of all members of the black hole hypermultiplet.

This result may have many interesting implications in quantum theory of black holes. We will argue in Appendix that virtual black holes and their superpartners may fix the values of the moduli fields in the vacuum. Whereas this idea is very speculative (that is why it is in Appendix), it deserves further investigation because it may provide a link between the supersymmetric models with symmetry breaking N = 2 → N = 1 and black hole physics.

## APPENDIX. IS IT POSSIBLE TO FIX SCALAR FIELDS BY VIRTUAL BLACK HOLES?

As we have found, not only black holes but their superpartners as well fix the values of the scalar fields on the horizon. Moreover, the masses of the black holes with given values of magnetic and electric charges become minimal if the value of the scalar field were equal to its value at the horizon in the whole universe. This particular value minimizes the black hole mass even if it is not extremal [3].

This makes it very tempting to consider only those configurations for which the total energy (mass) is minimal, and the scalar fields take the same values at the horizon and at infinity. Such configurations are called double-extreme [12]. They are particularly simple and allow much more detailed investigation than the black holes with scalar fields changing with the distance from a black hole.

Let us consider again the simplest example described in the Introduction. The scalar field (1) near the horizon approaches the asymptotic value (3), $e^{-2\phi} = \frac{|q|}{|p|}$. This asymptotic regime is reached only very close to the horizon. For example, for $p, q, \phi_0 \sim 1$ the dilaton stabilization occurs only at the Planckian distance from the horizon, $r < 1$; for larger $p$ and $q$ the asymptotic regime occurs earlier, at greater $r$.

The same property is shared by the multi-black-hole solutions describing a collection of extreme black holes with charges $cq$ and $cp$, where $c$ is any constant: Near each of the black holes the scalar field will approach the same value given by (3). If the black holes are very close to each other in some part of space, then the value of the dilaton field in this region will be close to (3) independently of $\phi_0$.

But do we actually need the gas of real black holes, or would it be enough to consider virtual black holes appearing in the vacuum and disappearing again? An important feature of the black hole attractors which is manifest in the simplest case considered in the Introduction is that the value of the dilaton field $\phi$ near the horizon of an extreme black hole does not depend on the sign of $q$ and $p$. It is also important that the masses of black holes (either extreme or non-extreme) have a minimum at the same value of $\phi$, independently of the sign of $q$ and $p$. Therefore both black holes and anti-black holes (black holes with opposite charges) will push the dilaton field to the same point (3), corresponding to the minimum of the effective mass $M(\phi)$ (2). Finally, in the previous section we have found that superpartners attract the dilaton field towards the same point. This makes it very tempting to speculate that black holes, their anti-particles and superpartners which exist as virtual states in the vacuum, may stabilize the values of moduli field in the vacuum, even in the absence of real black holes.

To make our idea more clear, suppose first that the only black holes that may appear in the vacuum are the ones with charges $\pm p$ and $\pm q$. Let us assume for a moment that their contribution to the effective potential is given by the standard one-loop expression for bose particles:

$$V(\phi) \sim \int d^4k \, \ln[k^2 + M^2(\phi)] \sim \int_0^\Lambda dk \, k^3 \ln[k^2 + M^2(\phi)] \;.$$

(36)

Here $\Lambda$ is the ultraviolet cut-off. Normally in quantum field theory one takes the limit $\Lambda \gg M$, calculates the integrals and makes the renormalization if necessary. In our case the situation is not that simple. In string theory there may be no momenta greater than $M_p = 1$. Meanwhile the mass of a black hole with large $p$ and $q$ is much greater than 1. Therefore the integral in (36) for large black holes should be calculated in a rather unusual limit $\Lambda \sim 1 \ll M$. This gives the following estimate for the dilaton effective potential induced by virtual black holes [15]:

$$V(\phi) \sim \int_0^1 dk \, k^3 \ln[k^2 + M^2(\phi)] \;\sim\; \ln M^2(\phi) \;. \quad (37)$$

Obviously, this effective potential has a minimum at the same point as the black hole mass $M(\phi)$ (2). Thus if our arguments are correct, nonperturbative effects associated with virtual black holes may stabilize the dilaton field at the same place at which the dilaton field is stabilized near the black hole horizon.

There are several problems associated with this proposal. Eq. (36) was written by analogy with the theory of point-like bose particles. However, black holes with non-vanishing entropy $S = 2\pi|pq|$ are not point-like objects, they are solitons which have finite size, and finite area of horizon and finite entropy. For large $p$ and $q$ they are surrounded by a large classical dilaton field, which in a certain sense can be considered a collection of many bose particles in the same state. If black holes were normal point-like particles described by quantum field theory, their contribution to $V(\phi)$ would be exactly cancelled by the contributions of their superpartners. One may expect that supersymmetry transformation transfers one of these bosons into a fermion. However, this does not necessarily mean that the contribution of such states to the effective potential must cancel the contribution of the original black holes carrying no fermions. To answer this question one should perform a real calculation of nonperturbative quantum effects induced by virtual black holes. In particular, one should take into account that non-extreme black holes may give a contribution to $V(\phi)$ as well, and this contribution will not be cancelled by the contribution of their superpartners because non-extreme black holes break supersymmetry. As we already emphasized, the mass $M(\phi)$ of non-extreme black holes with given $p, q$ and the entropy $S$ has a minimum with respect to $\phi$ at the same point as the mass of extreme black holes with the same $p$ and $q$. Therefore one may expect that if one takes the contribution of non-extreme black holes with the same absolute values of $p$ and $q$ and with the same entropy $S$, the resulting contribution will not be cancelled by the contribution of their superpartners. In this respect it is encouraging that the arguments presented above can get an additional support from our results concerning black hole superpartners. If one tries to visualize the vacuum state as a medium populated by virtual black holes and their superpartners, then from our results it follows that not only black holes but their superpartners as well attract the scalar fields to the same values. This makes it more plausible that virtual black holes and their superpartners may be responsible for fixing the values of the moduli fields in the vacuum.

On the other hand, the same reason which may produce a nonvanishing contribution of black holes to the effective potential (the non-perturbative nature of this effect) may imply that the effect in fact will be exponentially suppressed, just like any effects in the theory of instantons. Using thermodynamical analogy described in [16], one may expect that the "abundance" of virtual black holes in the vacuum should be suppressed by an exponential factor of the type of $e^{-2S} = e^{-4\pi|pq|}$ describing suppression of probability of a simultaneous production of a pair of black holes. This factor should be included in (36):

$$V(\phi) \sim e^{-2S(p,q)} \, F(M^2(\phi)) \; . \tag{38}$$

Here we have written some function $F(M^2(\phi))$ instead of $\ln M^2(\phi)$ to reflect the uncertainty of the estimates described above. We should emphasize that we used Coleman-Weinberg approach only as a heuristic way to estimate the effective potential $V(\phi)$. Fortunately, for the validity of our argument we do not really need to know the function $F(M^2(\phi))$ exactly. The only thing which is important to us is that if $F(M^2(\phi))$ is a monotonous function such as $M^2(\phi)$ or $\ln M^2(\phi)$ (37), then it has a minimum at the same point where $M^2(\phi)$ has a minimum, i.e. at $e^{-2\phi} = \frac{|q|}{|p|}$, see eq. (3). Thus one may argue that virtual extreme and non-extreme black holes can fix the dilaton field in vacuum at the same value at which extreme black holes fix this field near the horizon.

In the models describing N = 4 axion-dilaton black holes with electric and magnetic charges $n_1, n_2, m_1, m_2$ [5] the values of the dilaton field $\phi$ and the axion field $a$ at the extreme black hole horizon are given by

$$e^{-2\phi} = \frac{|n_2 \, m_1 - n_1 \, m_2|}{m_1^2 + m_2^2} \; , \qquad a = \frac{n_2 m_2 + n_1 m_1}{m_1^2 + m_2^2} \; . \tag{39}$$

As a result, one may expect that the sum of contributions of all virtual black holes with absolute values of electric and magnetic charges $|n_1|, |n_2|, |m_1|, |m_2|$ fixes the value of the dilaton field at the point (39). Note that this point which for the case $m_2 = 0$ corresponds to the minimum of the dilaton potential in the supersymmetric model of ref. [6]. This suggests that virtual black holes may provide a dynamical mechanism explaining a mysterious correspondence [5] between the behavior of scalar fields in new models of breaking of N = 2 SUSY down to N = 1 [6] and the properties of scalar fields near the horizon of the axion-dilaton black holes.

If one simultaneously changes sign of $n_i$ and $m_i$ (which should be allowed because we consider contributions of black holes and their anti-particles) the value of the axion field in (39) does not change. However, if there exist black holes with charges $(n_i, -m_i)$, then the field $a$ near the horizon of such black holes will change its sign. Therefore the possibility to fix the axion field in the vacuum requires a more careful analysis.

In a more general case one may consider vacuum populated by black holes belonging to several different hypermultiplets which are compatible with a given mechanism of stringy compactification. All such hypermultiplets may give contributions to the effective potential $V(\phi)$. Thus all of them should be taken into account, and one may need to take a sum over contributions of black holes with all possible values of $p$ and $q$. To find a complete expression for the effective potential one would need to perform a detailed calculation involving virtual black holes and their superpartners. Perhaps it would be more appropriate to study this issue in the context of D-brane theory. However, the calculation of the effective potential in this theory would go far beyond the scope of this paper. As a first step in this direction, one should establish relation between D-branes and fixed scalars. This will be a topic of a separate publication [17].

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